ℓ_1-norm regularized beamforming in ultrasound imaging

Teodora Szasz, Adrian Basarab, and Denis Kouamé
IRIT, UMR CNRS, University of Toulouse, Universit Paul Sabatier, France

***Abstract*—This paper is part of the challenge on plane wave imaging in medical ultrasound [1], organized during the IEEE International Ultrasonics Symposium 2016 in Tours (France). Herein, we address beamforming in ultrasound imaging, by formulating it, for each image depth, as an inverse problem solved using Laplacian prior through Basis Pursuit (BP). This approach was evaluated for the four different categories proposed for the competition, using 1, 11, 75 plane waves, and for the best ultrasound image quality using the lowest number of steered plane waves. The proposed method results in considerable improvement in spatial resolution and contrast compared with DAS method proposed in the challenge.**

***Index Terms*—Ultrasound imaging, beamforming, inverse problems, beamspace processing, Laplacian priors, Basis Pursuit.**

I. Introduction

The proposed method (see [2] and [3]), presents an ultrasound (US) beamforming (BF) technique using a linear inverse problem relating the raw data to the RF signals to be recovered. Numerical optimization routines have been employed to invert the resulting linear model based on standard regularization terms. ℓ_1-norm has been employed to generate the results presented through the challenge on plane wave imaging in medical ultrasound [1], organized during the IEEE International Ultrasonics Symposium 2016 in Tours (France). Our method was tested for the four categories of the challenge and the results will be presented in Section III.

Instead of using fixed (DAS) or adaptive (MV) apodization functions, we have recently proposed in [3] a new BF model for medical US imaging that takes into account the positions of the elements and reflectors in order to relate the desired signals to the raw data. This results in an inverse problem that we solve using Laplacian or Gaussian priors through Basis Pursuit (BP), respectively Least Squares methods. We were thus able to obtain two complementary results, one that produced sparse images and one generating smoother results [3]. This formulation also offered the possibility to highly reduce the number of US emissions during the scanning procedure, by integrating it with a beamspace processing technique [4]. Note that the same model was also applied in [5] and [6], but with different regularization functions.

This work uses the method presented in [3]. The direct model presented in Section II is inverted through Basis Pursuit (BP) using ℓ_1-norm regularization related to the Laplacian distributed nature of the images. This approach gives the best results in terms of the image quality metrics used in the challenge and visual perception of the images.

This work has been supported by the CIMI Labex, Toulouse, France, under grant ANR-11-LABX-0040-CIMI within the program ANR-11-IDEX-0002-02.

II. Proposed ℓ_1-norm regularized beamforming

A. Direct model formulation

Considering an M-element US probe that is sequentially transmitting P US plane-wave beams, let the recorded signals after time-of-flight delay compensation be of N time samples length. For the p^{th} emission and a given depth n, the model relating the received signal (raw data), $\boldsymbol{y}_p \in \mathbb{C}^{M\times 1}$ to the desired signal $\boldsymbol{x}$ can be written as follows:

$$\boldsymbol{y}_p = (\boldsymbol{A}_p^H \boldsymbol{A}_T)\boldsymbol{x} + \boldsymbol{g}_p, \tag{1}$$

where $\boldsymbol{A}_p \in \mathbb{C}^{M\times M}$ and $\boldsymbol{A}_T \in \mathbb{C}^{M\times K}$ are standard steering matrices relating the US probe element positions to the K lateral positions on the scanline. We denoted by $\boldsymbol{x} \in \mathbb{C}^{K\times 1}$ the signal at depth n to be beamformed with the proposed method, by $\boldsymbol{g}_p$ the additive white Gaussian noise affecting the raw data, and with $(\cdot)^H$ the conjugate transpose.

To reduce the dimensionality and increase the quality of the raw data, we apply beamspace processing, a common tool in array processing used to reduce the computational complexity and to improve the signal-to-noise ratio (SNR) [7]. In our case, we use the DAS BF result instead of the raw data, for transforming our data into beamspace domain. Thus, we can reformulate (1), as:

$$\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{g}, \tag{2}$$

where $\boldsymbol{\Phi} = \boldsymbol{A}^H \boldsymbol{A}_T$, $\boldsymbol{\Phi} \in \mathbb{C}^{M\times K}$, $\boldsymbol{y}$ is the DAS profile at depth n, x the signal of interest at depth n and g the Gaussian noise. Note that the resulting problem is ill-posed, thus requiring regularization in order to obtain a valid solution.

B. Model inversion

One solution, presented here, is to solve the BF problem in (2) by solving the following minimization problem:

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}}(\|\boldsymbol{y} - \boldsymbol{\Phi}\boldsymbol{x}\|_2^2 + \lambda\|\boldsymbol{x}\|_1), \tag{3}$$

where λ is the regularization parameter balancing the tradeoff between the fidelity to the data and the regularization term.

Herein, we used the well-known YALL1 to solve (3) [8], a software package dedicated to solve BP problems such as (3).

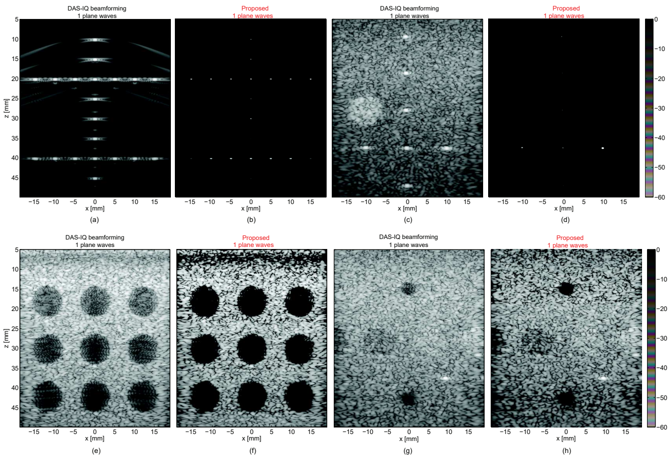

Fig. 1. Comparison of the proposed BF method ((b), (d), (f), (h)) with DAS BF ((a), (c), (e), (g)) for all challenge datasets, using 1 plane-wave.

TABLE I
MEAN RESOLUTION SCORES (AXIAL & LATERAL) AND CONTRAST SCORES (DB) IN FIG. 1

BF Method	Mean resolution (ax. & lat.)	Mean contrast (dB)
DAS - sim.	Fig. 1(a): 0.40 0.82	Fig. 1(e): 9.72
Proposed - sim.	Fig. 1(b): **0.06** **0.11**	Fig. 1(f): **12.10**
DAS - exp.	Fig. 1(c): 0.57 0.89	Fig. 1(g): 7.90
Proposed - exp.	Fig. 1(d): **0.05** **0.08**	Fig. 1(h): **10.40**

III. RESULTS AND DISCUSSION

This section presents the results obtained by our method on the datasets available through the challenge platform. Note that λ parameter was manually tuned to give the best numerical results.

A. *Results using 1 plane-wave*

Fig. 1 presents the results obtained with our method when 1 plane-wave was used.

B. *Results using the lowest number of steered plane-waves that results to the best image quality*

Hereafter, we present the results using three plane-waves that offer the best image quality metrics using the lowest number of steered plane-waves (see Fig. 2).

Table II presents the image quality metrics computed for Fig. 2.

C. *Results using 11 plane-waves*

Fig. 3 presents the results obtained with our method when 11 plane-waves were used.

Table III presents the image quality metrics computed for Fig. 3.

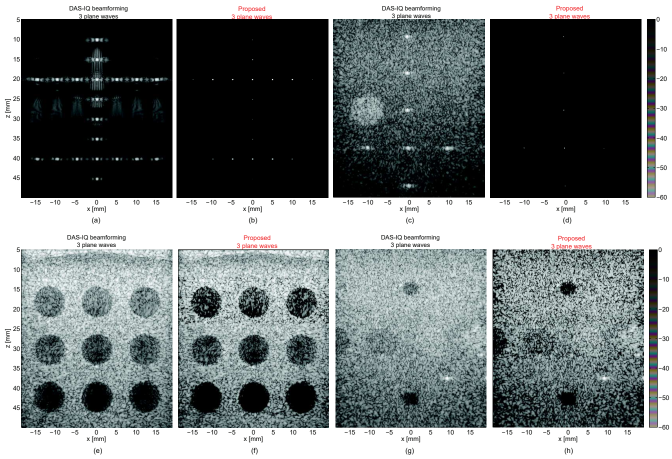

Fig. 2. Comparison of the proposed BF method ((b), (d), (f), (h)) with DAS BF ((a), (c), (e), (g)) for all challenge datasets, using 3 plane-waves.

TABLE II
MEAN RESOLUTION SCORES (AXIAL & LATERAL) AND CONTRAST SCORES (DB) IN FIG. 2

BF Method	Mean resolution (ax. & lat.)	Mean contrast (dB)
DAS - sim.	Fig. 2(a): 0.40 0.47	Fig. 2(e): 7.94
Proposed - sim.	Fig. 2(b): **0.13** **0.13**	Fig. 2(f): **8.56**
DAS - exp.	Fig. 2(c): 0.56 0.45	Fig. 2(g): 7.05
Proposed - exp.	Fig. 2(d): **0.11** **0.03**	Fig. 2(h): **8.60**

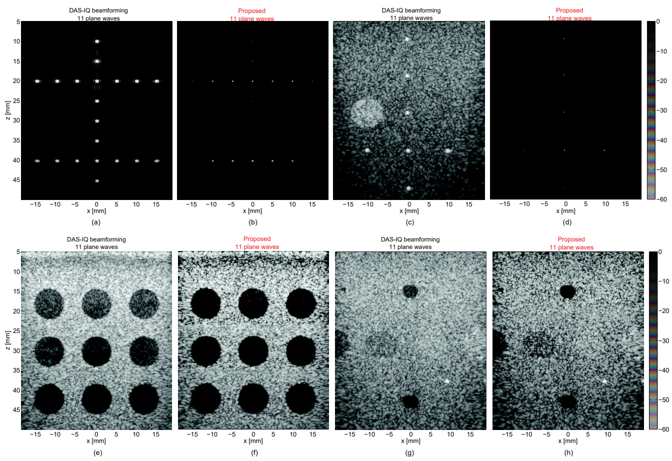

Fig. 3. Comparison of the proposed BF method ((b), (d), (f), (h)) with DAS BF ((a), (c), (e), (g)) for all challenge datasets, using 11 plane-waves.

TABLE III
MEAN RESOLUTION SCORES (AXIAL & LATERAL) AND CONTRAST SCORES (DB) IN FIG. 3

BF Method	Mean resolution (ax. & lat.)	Mean contrast (dB)
DAS - sim.	Fig. 3(a): 0.40 0.54	Fig. 3(e): 12.26
Proposed - sim.	Fig. 3(b): **0.11** **0.14**	Fig. 3(f): **15.52**
DAS - exp.	Fig. 3(c): 0.56 0.54	Fig. 3(g): 11.00
Proposed - exp.	Fig. 3(d): **0.09** **0.05**	Fig. 3(h): **14.15**

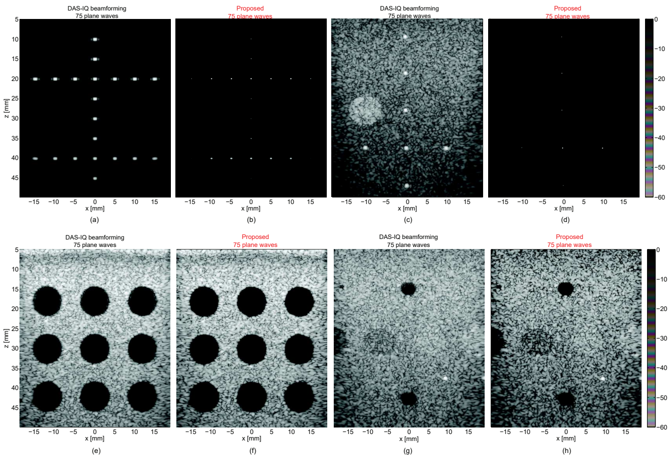

Fig. 4. Comparison of the proposed BF method ((b), (d), (f), (h)) with DAS BF ((a), (c), (e), (g)) for all challenge datasets, using 75 plane-waves.

TABLE IV
MEAN RESOLUTION SCORES (AXIAL & LATERAL) AND CONTRAST SCORES (DB) IN FIG. 4

BF Method	**Mean resolution (ax. & lat.)**	**Mean contrast (dB)**
DAS - sim.	Fig. 4(a): 0.40 0.56	Fig. 4(e): 15.36
Proposed - sim.	Fig. 4(b): **0.11** **0.15**	Fig. 4(f): **23.34**
DAS - exp.	Fig. 4(c): 0.56 0.56	Fig. 4(g): 11.80
Proposed - exp.	Fig. 4(d): **0.09** **0.05**	Fig. 4(h): **11.80**

D. Results using 75 plane-waves

Fig. 4 presents the results obtained with our method when 75 plane-waves were used.

Table IV presents the image quality metrics computed for Fig. 4.

In Fig. 1(d), Fig. 2(d), Fig. 3(d) and Fig. 4(d), the values of λ were manually chosen to provide the best values of the axial and lateral resolution. However, we can observe that the speckle and the cyst-like structure are eliminated. Note that the proposed method can also maintain the speckle and the cyst-like structure, by lowering the value of λ. An example, when using 1 plane-wave, is presented in Fig. 5. In Fig. 5(b) are conserved both the speckle, the cyst-like structure, while in Fig. 5(c), the speckle is eliminated. The values of λ and image quality metrics for each case can be found in Table V.

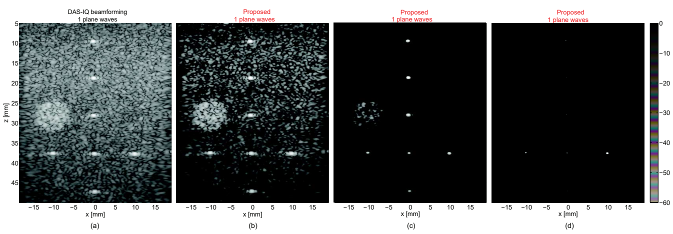

Fig. 5. The result of the proposed BF method for different value of λ, using 1 plane-wave. (a) DAS BF, (b),(c), (d), (e) proposed method with $\lambda = 10$, $\lambda = 50$, respectively $\lambda = 100$.

TABLE V
MEAN RESOLUTION SCORES (AXIAL & LATERAL)IN FIG. 5

BF Method	**Mean resolution (ax. & lat.)**
DAS.	Fig. 5(a): 0.57 0.89
Proposed ($\lambda = 10$)	Fig. 5(b): **0.53** **0.84**
Proposed ($\lambda = 50$)	Fig. 5(c): **0.35** **0.57**
Proposed ($\lambda = 100$)	Fig. 5(d): **0.05** **0.08**

IV. CONCLUSION

In this paper, we modeled US beamforming as an inverse problem regularized using ℓ_1-norm minimization. This formulation offers the possibility to integrate in the BF process the advantages of both sparse and smooth solutions. We have shown that our approach applied to the datasets available through the challenge platform provides solutions with improved axial and lateral resolution, and contrast compared to DAS.

REFERENCES

[1] "Beamforming grid | Plane-wave Imaging Challenge in Medical Ultra-Sound," .

[2] Teodora Szasz, Adrian Basarab, and Denis Kouamé, "Beamforming through regularized inverse problems in ultrasound medical imaging," *arXiv:1507.08184 [cs]*, July 2015, arXiv: 1507.08184.

[3] Teodora Szasz, Adrian Basarab, and Denis Kouam, "Beamforming through regularized inverse problems in ultrasound medical imaging (to appear)," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control*, 2016.

[4] J.J. Fuchs, "Linear programming in spectral estimation. Application to array processing," in *, 1996 IEEE International Conference on Acoustics, Speech, and Signal Processing, 1996. ICASSP-96. Conference Proceedings*, May 1996, vol. 6, pp. 3161–3164 vol. 6.

[5] T. Szasz, A. Basarab, M. F. Vaida, and D. Kouam, "Elastic-net based beamforming in medical ultrasound imaging," in *2016 IEEE 13th International Symposium on Biomedical Imaging (ISBI)*, Apr. 2016, pp. 477–480.

[6] Teodora Szasz, Adrian Basarab, and Denis Kouam, "Beamforming of ultrasound images modelled as stable random variables (regular paper)," in *IEEE International Ultrasonics Symposium, Tours, France, 18/09/2016-21/09/2016*, 2016.

[7] D. Malioutov, M. Cetin, and A.S. Willsky, "A sparse signal reconstruction perspective for source localization with sensor arrays," *IEEE Transactions on Signal Processing*, vol. 53, no. 8, pp. 3010–3022, Aug. 2005.

[8] Yin Zhang, "User-guide for yall1: Your algorithms for l1 optimization," 2009.